Library, the Franklin Institute, the Penn Monthly, the Librarian of Congress, the Chief of U. S. Engineers, Senator Charles Sumner, and Prof. F. V. Hayden.

The death of Christian Olrik, of Denmark, a member of the Society, was announced by the Secretary.

The committee to which was referred the paper of Prof. Stevenson, on West Virginia Coal Measures, reported in favor of its publication in the Transactions, which on motion was so ordered.

A communication entitled, On some remarkable relations between the mean motions of Jupiter, Saturn, Uranus, and Neptune, received by letter from Prof. Daniel Kirkwood, dated Bloomington, Monroe County, Ind., March 11, 1872, was read by the Secretary.

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## On some Remarkable Relations between the Mean Motions of Jupiter, Saturn, Uranus, and Neptune.

By Prof. Daniel Kirkwood.

(*Read before the American Philosophical Society, March 15th*, 1872.)

It was noticed by the writer several years since, that 85 periods of Jupiter are nearly equal to 12 of Uranus,* and that 149 periods of Uranus are approximately equal to 76 of Neptune. If, therefore, $n^{v}$, $n^{vi}$, $n^{vii}$ and $n^{viii}$ denote the respective mean motions of Jupiter, Saturn, Uranus, and Neptune, these relations are expressed as follows:

$$12n^{v} - 85^{vii} = 76^{vii} - 149^{viii}, \text{ nearly};$$

or,
$$12n^{v} - 161n^{vii} + 149n^{viii} = -\gamma \quad . \quad . \quad . \quad . \quad (1).$$

With Newcomb's value of $n^{viii}$ and the values of $n^{v}$ and $n^{vii}$ adopted in the *American Ephemeris*, we find $\gamma = 390''$. The equation,

$$32n^{vi} - 153n^{vii} + 121n^{viii} = -\gamma \quad . \quad . \quad . \quad (2),$$

was obtained by a process somewhat similar. Subtracting (2) from (1), and dividing by 4, we have

$$3n^{v} - 8n^{vi} - 2n^{vii} + 7n^{viii} = 0, \quad . \quad . \quad . \quad . \quad (3).\dagger$$

This equation, like that which exists between the mean motions of Jupiter's first three satellites, is doubtless exact. The mean motion of Neptune is less accurately known than those of the old planets. If we assume, then, that the received values of $n^{v}$, $n^{vi}$, $n^{vii}$, are correct,

* Runkle's Mathematical Monthly, January, 1860.

† Equation (3), without any account of its discovery, was given in Silliman's Journal, March, 1872.

the value of $n_{\text{viii}}$, found by equation (3), is 7863″.983, differing from Newcomb's value (7864″.935) by less than 1″.

The corresponding relations between the *mean longitudes* of the four outer planets are sufficiently obvious. Thus,

$$3l^{\text{v}} - 8l^{\text{vi}} - 2l^{\text{vii}} + 7l^{\text{viii}} = \chi = \text{a constant} \quad . \quad . \quad . \quad (4).$$

With a slight correction of the elements, it will probably be found that $\chi = 135°$.

Again: the equation,

$$17n^{\text{vi}} - 228n^{\text{vii}} + 211n^{\text{viii}} = 0, \quad . \quad . \quad . \quad . \quad (5).$$

found in the same manner as (1), is believed to be exact. Combining (3) and (5), we obtain

$$68n^{\text{vi}} - 325n^{\text{vii}} + 257n^{\text{viii}} = 0, \quad . \quad . \quad . \quad (6),$$
$$257n^{\text{v}} - 844n^{\text{vi}} + 587n^{\text{vii}} = 0, \quad . \quad . \quad . \quad (7),$$
$$325n^{\text{v}} - 912n^{\text{vi}} + 587n^{\text{viii}} = 0, \quad . \quad . \quad . \quad (6).$$

These equations indicate that in a cycle of about 11657.2969 Julian years, the planets Jupiter, Saturn, Uranus, and Neptune return to the same relative mean longitudes. The equations are all satisfied by the following values. The received values are given for the convenience of comparison. In column first $\delta'' = \frac{1296000''}{11657.2969}$.

| THEORETICAL VALUES. | | RECEIVED VALUES. | DIFFERENCES. |
|---|---|---|---|
| $n^{\text{v}}$ | $= 109256''.719$ | 109256″.719 | 0″.000 |
| $n^{\text{vi}} = n^{\text{v}} - 587\delta'' =$ | 43996.971 | 43996.127 | +0″.844 |
| $n^{\text{vii}} = n^{\text{v}} - 844\delta'' =$ | 15424.986 | 15424.509 | +0″.477 |
| $n^{\text{viii}} = n^{\text{v}} - 912\delta'' =$ | 7865.083 | 7864.935 | +0″.148 |

The received value of Jupiter's mean motion is here assumed to be correct. Any change would produce a corresponding variation in the remaining values. A revision of the theory of the orbits will, of course, result in some slight modifications. I believe, however, that the relations expressed by the preceding equations will be found strictly exact. If so, it must follow that *no three of the four outer planets can ever be in conjunction at the same time.*

BLOOMINGTON, IND., *February*, 1872.